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Nonlinear chiral σ model for nuclear matter

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In order to include pionic degrees of freedom in the description of nuclear many-body systems, the chiral σ model in the nonlinear representation is investigated. The renormalizability of the model, which is obtained from the linear σ model by a field transformation, is studied in the context of the equivalence theorem. It is shown that in any expansion scheme which is based on self-consistent mean scalar fields, the nonlinear σ model should be considered as unrenormalizable (even if the σ mass is kept finite), and new counterterms have to be introduced in each order. The resulting equation of state in the one-loop (Hartree) approximation is calculated, and the corresponding pion-nucleus optical potential is discussed. [S0556-2813(97)03209-3]

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I. INTRODUCTION

One of the most important subjects in relativistic nuclear many-body theories is to assess the effects of vacuum fluctuations, which cannot be studied in nonrelativistic models. For this purpose, many works used some kind of meson-nucleon theory incorporating scalar (σ) and neutral vector (ω) meson field [1]. On the other hand, nonrelativistic models have clearly demonstrated the importance of pionic degrees of freedom [2], in particular for the description of nuclear electroweak properties [3]. It is therefore desirable to devise chiral-invariant relativistic models which are capable of including both pionic degrees of freedom and vacuum fluctuation effects at the same time.

To investigate the effects of vacuum fluctuations, it seems natural to use a renormalizable model, where the calculations can in principle be done in a well-defined and straightforward way. Therefore, the chiral linear σ model [4] was considered as a possible candidate. However, it was realized soon that this model leads to serious difficulties: As a result of the nonderivative coupling of the pion, intricate cancellations are inevitable in order to produce results consistent with low-energy theorems and current algebra. In the nuclear medium, this leads to the difficulty that in the mean-field (Hartree) approximation the pion propagator has a tachyon pole [5,6] (i.e., a pole at imaginary energy).[1] Explicit calculations have shown that higher order loop graphs of the ring type are actually insufficient to prevent this tachyon pole [6]. On the other hand, in the nonlinear representation [7] the pion couples via derivatives, and no cancellations are necessary to reproduce the results of low-energy theorems. For example, concerning the tachyon pole problem mentioned above, the effective pion mass in the nonlinear representation is of order m_π^2 from the outset and positive definite. Therefore, the σ model in the nonlinear representation seems to be a better starting point.

Although the nonlinear σ model is unrenormalizable by power counting, its Lagrangian can be obtained from the linear model by a field transformation (Weinberg transformation) [8]. Because of the equivalence of Lagrangians which are connected by nonlinear field transformations [9–11], one might therefore expect [12,13] that the nonlinear model is still "on-shell renormalizable" (as long as the σ meson mass is kept finite), although the usual multiplicative renormalizability is lost. Indeed, calculations including one-nucleon loop graphs [12] (but no meson loops) seemed to support this conjecture, and the viewpoint that the nonlinear model should lead to finite physical results if the counterterms of the linear model are included in the transformation can be found sometimes in the literature [13,14]. In view of this fact and the recent interest in linear and nonlinear σ models for nuclear systems, often in connection with the concept of scale invariance [15], it seems necessary to us to clarify in which sense the nonlinear model is equivalent to the linear one, in particular for the description of the bulk properties of nuclear matter. This is one of the main purposes of this work. We will show in detail that results for the physical quantities in the two theories, calculated up to some order in an expansion scheme, are equivalent only if the mean scalar fields in the two theories are taken into account up to the same order. In any treatment which is based on *self-consistent* mean scalar fields, however, the equivalence is lost. Concrete one-loop examples will be used to illustrate this feature. Consequently, in these treatments the nonlinear model should be considered as unrenormalizable, and new counterterms are necessary in each order of the expansion.

Following this procedure, we will calculate the thermodynamic potential and the resulting equation of state (EOS) of nuclear matter in the nonlinear model to one-loop order. This requires the introduction of a new counterterm to renormalize the pion loop contribution, i.e., the shift of the pionic zero-point energy relative to the vacuum value, which arises due to an enhanced pion mass in the medium. This enhanced pion mass, which originates from the s-wave pion-nucleon

[1]This problem does not appear in an "exact" calculation, but shows up in any expansion scheme of the effective potential to some order ℓ, which requires the pion propagator up to order $\ell - 1$ as an input. (See the discussion at the end of Sec. II.)

interaction, has only a little effect on the bulk properties, and we will explore some consequences of it for the pion-nucleus optical potential. The main purpose of this paper, however, is to provide a basis for an application of the nonlinear σ model to the description of nuclear matter, and the numerical results presented here serve as illustrative examples.

The rest of the paper is organized as follows. In Sec. II the linear σ model and problems associated with it are discussed. In Sec. III the Lagrangian is transformed to the nonlinear representation, and it is shown that the pion loop contribution to the effective potential cannot be renormalized by the counterterms present in the original Lagrangian. In Sec. IV the equivalence theorem is reviewed and applied to the present problem. In Sec. V a new counterterm is introduced to renormalize the pion loop contribution, the resulting one-loop equation of state at finite temperature is calculated, and the pion-nucleus optical potential is discussed. Conclusions are presented in Sec. VI.

II. LINEAR σ MODEL

The Lagrangian of the chiral linear σ model is given by [4]

$$\mathcal{L}=\bar{\psi}[i\partial\!\!\!/-g(\phi+i\boldsymbol{\pi}\cdot\boldsymbol{\tau}\gamma_5)]\psi+\frac{1}{2}[(\partial_\mu\phi)^2+(\partial_\mu\boldsymbol{\pi})^2] -\frac{\mu^2}{2}(\phi^2+\boldsymbol{\pi}^2)-\frac{\lambda^2}{4}(\phi^2+\boldsymbol{\pi}^2)^2+c\phi. \tag{2.1}$$

Shifting $\phi=u+\sigma$, we obtain

$$\hat{\mathcal{L}}=-V_{\rm cl}(u)+\bar{\psi}[i\partial\!\!\!/-m_N(u)-g(\sigma+i\boldsymbol{\pi}\cdot\boldsymbol{\tau}\gamma_5)]\psi +\frac{1}{2}[(\partial_\mu\sigma)^2-m_\sigma^2(u)\sigma^2]+\frac{1}{2}[(\partial_\mu\boldsymbol{\pi})^2-m_\pi^2(u)\boldsymbol{\pi}^2] -\lambda^2u\sigma(\sigma^2+\boldsymbol{\pi}^2)-\frac{\lambda^2}{4}(\sigma^2+\boldsymbol{\pi}^2)^2+\sigma[c-um_\pi^2(u)], \tag{2.2}$$

where

$$V_{\rm cl}(u)=\frac{\lambda^2}{4}u^4+\frac{\mu^2}{2}u^2-cu$$

is the classical potential, and $m_N(u)=gu$, $m_\pi^2(u)=\mu^2+\lambda^2u^2$, and $m_\sigma^2(u)=\mu^2+3\lambda^2u^2$. The "physical" value of u (i.e., the thermal average, or the vacuum expectation value in the case of $T=0$, of ϕ) will be denoted by v, and can be obtained from the condition [4] that the linear term in Eq. (2.2) cancels the σ tadpole loop graphs $[S(u)]$:

$$J(u)\equiv c-u[m_\pi^2(u)+\delta m_\pi^2(u)]-iS(u)=0 \quad \text{at } u=v. \tag{2.3}$$

The counterterms are obtained by replacing $\mu^2\rightarrow\mu^2+\delta\mu^2$ and $\lambda^2\rightarrow\lambda^2+\delta\lambda^2$, besides the wave function renormalizations and the renormalization of g. This gives a contribution $\propto\delta m_\pi^2(u)=\delta\mu^2+\delta\lambda^2u^2$ in the linear term of Eq. (2.2), which has been included in Eq. (2.3). The particle masses at $u=v$ will be denoted by $m_N\equiv m_N(v)$ and $m_\alpha^2\equiv m_\alpha^2(v)$ $(\alpha=\sigma,\pi)$. Note that, since v depends on temperature and density, these are in-medium masses. Their relation to the free (subscript f) masses is $m_N=(v/v_f)m_{Nf}$, and

$$m_\alpha^2=m_{\alpha f}^2+n_\alpha\lambda^2(v^2-v_f^2) \quad (\alpha=\sigma,\pi,\ n_\sigma=3,\ n_\pi=1). \tag{2.4}$$

The calculation of the one-loop thermodynamic potential using the imaginary-time path integral is standard [16]: One adds a source term ϕJ and a chemical potential term $\mu_c\bar{\psi}\gamma^0\psi$ to the Lagrangian (2.1) and represents the partition function $Z(J)=\exp(i/\hbar)W(J)$ by a path integral, where the t integration in the classical action is evaluated along a straight line (C) from $t=0$ to $t=-i\beta$. One then performs a Legendre transformation to the effective action $\Gamma(u)\equiv i(\beta V)V_{\rm eff}=W(J)-\int d^4xJ\phi$. This leads to

$$V_{\rm eff}(u)=\frac{-\hbar}{\beta V}\ln\left[\int\mathcal{D}\psi\mathcal{D}\bar{\psi}\mathcal{D}\sigma\mathcal{D}\boldsymbol{\pi}\times\exp\left(\frac{i}{\hbar}\int_C d^4x(\hat{\mathcal{L}}+J\sigma+\mu_c\bar{\psi}\gamma^0\psi)\right)\Bigg|_{J=J(u)}\right], \tag{2.5}$$

where $J(u)=-\partial V_{\rm eff}/\partial u$ has the form (2.3). The thermodynamic potential per unit volume is then $\Omega/V=V_{\rm eff}(v)$. To $O(\hbar)$ one obtains [5,17]

$$V_{\rm eff}(u)=V_{\rm cl}(u)+\delta V_{\rm cl}(u)+i\hbar\sum_{\alpha=N,\sigma,\pi}\int\frac{d^4k}{(2\pi)^4}c_\alpha\times\ln[k^2-m_\alpha^2(u)+i\epsilon]+V_D(u) \equiv V_{\rm cl}+V_F+V_D, \tag{2.6}$$

where $c_N=4$, $c_\sigma=-\frac{1}{2}$, $c_\pi=-\frac{3}{2}$,

$$\delta V_{\rm cl}(u)=\frac{\delta\lambda^2}{4}u^4+\frac{\delta\mu^2}{2}u^2$$

is the counterterm to $O(\hbar)$, and V_D is finite and vanishes as μ_c, $T\rightarrow0$.

Using dimensional regularization we have [17]

$$i\int\frac{d^4k}{(2\pi)^4}\ln[k^2-m_\alpha^2(u)+i\epsilon]=\frac{m_\alpha^4(u)}{32\pi^2}[K-\ln m_\alpha^2(u)], \tag{2.7}$$

where $K=1/\epsilon_D-\gamma+\ln4\pi+\frac{3}{2}$ and $\epsilon_D=2-D/2$. From the above forms for $m_\alpha^2(u)$ we see that all divergences can be put into $\delta\lambda^2$ and $\delta\mu^2$. To fix the finite parts of these constants, one usually requires that for $T=\mu_c=0$ there be no loop contributions to the one- and two-point σ Green functions at zero momentum [1,5], i.e., that $\partial V_F/\partial u|_{u=v_f}=\partial^2V_F/\partial u^2|_{u=v_f}=0$. [The first condition means that in the vacuum the counterterm in Eq. (2.3) cancels the loop graphs S, such that $m_{\pi f}^2=c/v_f$.] The explicit form of V_F can be found in [5].

From Eq. (2.4) we see that for fixed $m_{\pi f}^2$, the in-medium mass squared m_π^2 becomes negative if $v<v_f$, even for slight deviations of v from v_f. This is the tachyon pole problem discussed in the Introduction.[2] It should be emphasized that in a "full" calculation this problem does not appear due to the Goldstone theorem in the medium [5], which states that $iS(v)=v\bar{\Sigma}_\pi(0)$, where $\bar{\Sigma}_\pi(0)$ is the unrenormalized pion self-energy in the linear model at $k=0$, i.e.,

$$\frac{\partial V_{\rm eff}}{\partial u}=-u\Delta_\pi^{-1}(0)-c=0 \quad (\text{at } u=v). \tag{2.8}$$

Here $-\Delta_\pi^{-1}(0)=m_\pi^2+\delta m_\pi^2+\bar{\Sigma}_\pi(0)$. Therefore, at $u=v$, where the effective potential has its minimum, the inverse pion propagator at zero momentum becomes $-\Delta_\pi^{-1}(0)=c/v>0$. In a loop expansion (or any other expansion scheme), however, the effective potential to order ℓ ($V_{\rm eff}^{(\ell)}$) involves the pion propagator up to order $\ell-1$, and nothing prevents $-\Delta_\pi^{-1(n)}(0)$ $(n=0,1,\ldots,\ell-1)$ from becoming negative, when evaluated at the minimum of $V_{\rm eff}^{(\ell)}$. In our example above, the one-loop effective potential involves the integral over the zero-loop (Hartree) pion propagator, which indeed has no tachyon pole at the "zero-loop value" $v=v_f$, but becomes tachyonic for any other $v<v_f$. Accordingly, $V_{\rm eff}^{(\ell)}$ becomes complex, and a loop expansion of the effective potential in the linear σ model is rendered impossible.

III. WEINBERG TRANSFORMATION

To obtain the σ model Lagrangian in the nonlinear representation of chiral symmetry, one can introduce the chiral radius (ϕ') and the chiral angle ($\boldsymbol{\theta}$) instead of the original variables $\phi,\boldsymbol{\pi}$, i.e., $\phi+i\boldsymbol{\pi}\cdot\boldsymbol{\tau}\gamma_5=\phi' U$ with $U=\exp(i\boldsymbol{\theta}\cdot\boldsymbol{\tau}\gamma_5)$. One also performs a chiral rotation of the nucleon field [8] $N=U^{1/2}\psi$, such that the meson-nucleon interaction part of Eq. (2.1) becomes simply $-g\bar{N}\phi' N$. Instead of the chiral angle $\boldsymbol{\theta}$, however, it is customary to use the variable $\boldsymbol{\xi}=\hat{\boldsymbol{\theta}}\tan(\theta/2)$, in terms of which $U^{1/2}$ takes the form $U^{1/2}=(1+i\boldsymbol{\xi}\cdot\boldsymbol{\tau}\gamma_5)/(1+\xi^2)^{1/2}$. As a result, the transformation $(\phi,\boldsymbol{\pi},\psi)\to(\phi',\boldsymbol{\xi},N)$ is given by [8,13]

$$\phi=\frac{1-\xi^2}{1+\xi^2}\phi', \quad \boldsymbol{\pi}=\frac{2\boldsymbol{\xi}}{1+\xi^2}\phi', \quad \psi=\frac{1-i\boldsymbol{\xi}\cdot\boldsymbol{\tau}\gamma_5}{(1+\xi^2)^{1/2}}N. \tag{3.1}$$

The Jacobian of this transformation is given by

$$J=\det(A_{ij}), \quad A_{ij}=\frac{2\phi'}{1+\xi^2}\delta_{ij}\equiv A\,\delta_{ij}. \tag{3.2}$$

[2]For $v\lesssim v_f/\sqrt{3}$, the σ meson becomes tachyonic, too. The calculations (see Sec. V), however, show that this presents no problem as long as one is not interested in the transition to the abnormal state ($v=0$). Moreover, it is known that this problem can be solved at least partially by including the ring-type diagrams [17–19].

The det in Eq. (3.2) refers to space-time as well as to the isospin indices i,j. Inserting Eq. (3.1) into Eq. (2.1), and introducing ghost fields η,η^* to account for the Jacobian (3.2), one arrives at the Lagrangian of the chiral nonlinear σ model:

$$\begin{aligned}\mathcal{L}'=&\bar{N}[i\not{D}-g\phi'+(D^\mu\boldsymbol{\xi})\cdot\boldsymbol{\tau}\gamma_\mu\gamma_5]N\\&+\frac{1}{2}[(\partial_\mu\phi')^2+4\phi'^2(D^\mu\boldsymbol{\xi})^2]-\frac{\mu^2}{2}\phi'^2\\&-\frac{\lambda^2}{4}\phi'^4+c\phi'\frac{1-\xi^2}{1+\xi^2}+\eta\eta^*\left(\frac{2\phi'}{1+\xi^2}\right)^3.\end{aligned} \tag{3.3}$$

Here the covariant derivatives are defined by [7] $D^\mu\boldsymbol{\xi}=[1/(1+\xi^2)](\partial^\mu\boldsymbol{\xi})$ and $D^\mu N=[\partial^\mu+i\boldsymbol{\tau}\cdot(\boldsymbol{\xi}\times D^\mu\boldsymbol{\xi})]N$.

The chiral transformation properties of the new fields and the role of the Jacobian are discussed in Appendix A. As is argued there, the Jacobian factor should be included even if one starts directly from the nonlinear σ model, although we derived it here from the field transformations.

We now shift the new scalar field $\phi'=u+\sigma'$. It is also convenient to introduce a pion field $\boldsymbol{\pi}'=2u\boldsymbol{\xi}$ such that the factor in front of the kinetic term becomes $\frac{1}{2}$. Of course, the determinant changes accordingly, and we obtain the shifted Lagrangian of the nonlinear model [12]:

$$\begin{aligned}\hat{\mathcal{L}}'=&-V_{\rm cl}(u)+\bar{N}\left[i\not{D}-m_N(u)-g\sigma'\right.\\&\left.+\frac{1}{2u}(D^\mu\boldsymbol{\pi}')\cdot\boldsymbol{\tau}\gamma_\mu\gamma_5\right]N+\frac{1}{2}[(\partial_\mu\sigma')^2-m_\sigma^2(u)\sigma'^2]\\&+\frac{1}{2}\left[\left(1+\frac{\sigma'}{u}\right)^2(D^\mu\boldsymbol{\pi}')^2-\mu_\pi^2(u)\frac{\boldsymbol{\pi}'^2}{1+\boldsymbol{\pi}'^2/4u^2}\right]\\&-\lambda^2u\sigma'^3-\frac{\lambda^2}{4}\sigma'^4-\frac{1}{2}\mu_\pi^2\frac{\sigma'}{u}\frac{\boldsymbol{\pi}'^2}{1+\boldsymbol{\pi}'^2/4u^2}\\&+\eta\eta^*\left(\frac{1+\sigma'/u}{1+\boldsymbol{\pi}'^2/4u^2}\right)^3+\sigma'[c-um_\pi^2(u)].\end{aligned} \tag{3.4}$$

Here the covariant derivatives are given by

$$D^\mu N=\left[\partial^\mu+i\frac{1}{(2u)^2}\boldsymbol{\tau}\cdot(\boldsymbol{\pi}'\times D^\mu\boldsymbol{\pi}')\right]N,$$

$$D^\mu\boldsymbol{\pi}'=\frac{1}{1+\boldsymbol{\pi}'^2/4u^2}(\partial^\mu\boldsymbol{\pi}'). \tag{3.5}$$

The quantities $m_N(u)$, $m_\sigma^2(u)$, and also $m_\pi^2(u)$ in the term $\propto\sigma'$ in Eq. (3.4), are the same as those in Eq. (2.2), but the new in-medium pion mass squared $\mu_\pi^2(u)=c/u$ is a positive definite quantity. The physical value of u for the nonlinear model, i.e., the thermal average of ϕ', will be denoted by v'. It is different from v of the linear theory, as is clear from the form of the transformation (3.1). It can be obtained from the condition that the linear term in Eq. (3.4) cancels the tadpole graphs $[S'(u)]$ of the nonlinear model:

$$J'(u) \equiv c - u[m_\pi^2(u) + \delta m_\pi^2(u)] - iS'(u) = 0 \quad \text{at } u = v' . \tag{3.6}$$

The effective nucleon and σ masses are then $m_N' \equiv m_N(v')$ and $m_\sigma'^2 \equiv m_\sigma^2(v')$. The square of the effective pion mass in the nonlinear model,

$$\mu_\pi'^2 \equiv \mu_\pi^2(v') = \frac{c}{v'} = \mu_{\pi f}^2 \frac{v_f'}{v'} , \tag{3.7}$$

has the same dependence on the scalar mean field as the full inverse propagator $[-\Delta_\pi^{-1}(0)]$ had in the linear model; see Eq. (2.8).

If the counterterms of the linear model are included in the transformation, it is clear that we have to replace $\mu^2 \to \mu^2 + \delta\mu^2$ and $\lambda^2 \to \lambda^2 + \delta\lambda^2$ everywhere in Eq. (3.4), besides the wave function renormalizations and the renormalization of g. This leads, in particular, to the counterterm $\propto \delta m_\pi^2(u)$ for tadpole graphs in Eq. (3.6). There is, however, no pion mass counterterm in the transformed (nonlinear) Lagrangian.

Let us now discuss the one-loop effective potential in the nonlinear model. It is clear that the integration over the ghost fields in the effective potential [cf. Eq. (2.5)] gives back the logarithm of the Jacobi determinant, i.e., a contribution

$$\propto \hbar \mathrm{Tr} \ln \left(\frac{1 + \sigma'/u}{1 + \boldsymbol{\pi}'^2/4u^2} \right) ,$$

to be integrated over the meson fields. Since the integrand involves only the fluctuation fields, it is clear that the ghost loop gives contributions of order $\hbar^2$ and higher. Therefore our effective potential takes again the form (2.6), where the effective masses have to be replaced by those of the nonlinear model. The nucleon and the σ loop are formally unchanged, while the pion loop becomes

$$\bar{V}_{F,\pi} = -\frac{3i}{2}\hbar \int \frac{d^4k}{(2\pi)^4} \ln[k^2 - \mu_\pi^2(u) + i\epsilon]$$
$$= -\frac{3\hbar}{64\pi^2} \frac{c^2}{u^2} [K - \ln \mu_\pi^2(u)] . \tag{3.8}$$

Because of the inverse power dependence on u, this divergence cannot be canceled by the term $\delta V_{\mathrm{cl}}(u)$ in Eq. (2.6). (We note that this feature is independent of the choice of the normalization of the field $\boldsymbol{\pi}'$, as long as the Jacobian is properly adjusted.) In other words, the counterterms of the linear model are insufficient to make the effective potential in the nonlinear model finite.

A more general discussion concerning this point will be given in the next section. Here we just note that we should compare the effective potentials in the two theories at their respective "physical" values $u = v$ and $u = v'$ [see Eq. (4.4) below]. If these quantities are determined self-consistently from Eqs. (2.3) and (3.6), they implicitly include all orders of $\hbar$. The "pure $O(\hbar)$" effective potential is obtained by inserting the $O(\hbar^0)$ solution, i.e., the minimum of $V_{\mathrm{cl}}(u)$, into the $O(\hbar)$ parts of the effective potential. This $O(\hbar^0)$ solution is independent of T, ρ and is the same in both models $(v_f = v_f')$. Since for this solution $m_\pi^2(v_f) = \mu_\pi^2(v_f) = c/v_f$, the "pure $O(\hbar)$" parts *are* identical in both theories.[3] (This is, of course, of little practical importance, since these parts are removed by the subtraction of the vacuum values.) This observation indicates already that the loop expansion in the two theories are equivalent to each other only if one really compares terms of the same order in $\hbar$, taking into account also the $\hbar$ dependence of the mean fields v and v'.

IV. RELATION BETWEEN THE LINEAR AND NONLINEAR σ MODEL

In this section we first consider some aspects of the equivalence theorem which are relevant to understand the connection between the linear and nonlinear σ model. We then explain why in actual loop calculations based on self-consistent mean fields the nonlinear σ model should be considered as unrenormalizable.

A. Equivalence theorem and one-loop examples

Our notation will, for simplicity, refer to a single scalar field (ϕ), but the generalization will be obvious. We consider a canonical transformation of field variables $\phi = \phi[\phi']$ such that $\mathcal{L}(\phi) = \mathcal{L}(\phi[\phi']) \equiv \mathcal{L}'(\phi')$. We also introduce the shifts $\phi = \sigma + u$ and $\phi' = \sigma' + u'$, and denote the shifted Lagrangians by $\hat{\mathcal{L}}(\sigma, u) \equiv \mathcal{L}(\sigma + u)$ and $\hat{\mathcal{L}}'(\sigma', u') \equiv \mathcal{L}'(\sigma' + u')$. The physical values of u and u' in the two theories are denoted as v and v'. Then the connection (or equivalence) between the original and the transformed theories can be summarized as follows [9–11].

(1) If $\langle \phi\phi \cdots \rangle^{\mathcal{L}}$ denotes the average of $\phi(x_1)\phi(x_2)\cdots$ with weight $\exp(i/\hbar)\int d^4x \mathcal{L}$, we have

$$\langle \phi\phi \cdots \rangle^{\mathcal{L}(\phi)} = \langle \phi[\phi']\phi[\phi'] \cdots \rangle^{\mathcal{L}'(\phi')} , \tag{4.1}$$

$$\langle \sigma\sigma \cdots \rangle^{\hat{\mathcal{L}}(\sigma,v)} = \langle \sigma[\sigma',v']\sigma'[\sigma',v'] \cdots \rangle^{\hat{\mathcal{L}}'(\sigma',v')} . \tag{4.2}$$

The first relation applied for the one-point function gives the relation between the mean fields, $v = v(v')$, and therefore the shifted fields are related by

$$\sigma[\sigma', v'] = \phi[\sigma' + v'] - v(v') , \tag{4.3}$$

which is used in Eq. (4.2). Note that there is no simple relation between the n-point functions in the two theories, i.e., between $\langle \phi\phi \cdots \rangle^{\mathcal{L}(\phi)}$ and $\langle \phi'\phi' \cdots \rangle^{\mathcal{L}'(\phi')}$, or similarly between the shifted n-point functions. In particular, if the n-point functions in the original theory are made finite by

[3]This is evident for the loop parts of the effective potential. From the classical potential in the transformed theory there also arises an additional $O(\hbar)$ term due to $\Delta v \equiv v' - v \propto \hbar$, i.e., $V_{\mathrm{cl}}(v') = V_{\mathrm{cl}}(v) + \Delta v[(\lambda^2 v_f^2 + \mu^2)v_f - c] + O(\hbar^2)$. However, the term in brackets $[\cdots]$ vanishes since v_f minimizes $V_{\mathrm{cl}}(u)$.

renormalization, the n-point functions of the new theory will nevertheless be divergent in general [10].

(2) The effective actions of the two theories at their respective minima are the same:

$$\Gamma(v)=\Gamma'(v'). \tag{4.4}$$

Relation (4.4) shows in particular that the thermodynamic potential and, therefore, all bulk properties of the system are the same in the two theories.

(3) The two-point functions[4] $\Delta(p)=\langle\sigma(p)\sigma(-p)\rangle^{\hat{\mathcal{L}}(\sigma,v)}$ and $\Delta'(p)=\langle\sigma'(p)\sigma'(-p)\rangle^{\hat{\mathcal{L}}'(\sigma',v')}$ have the same pole; i.e., at the pole they behave as

$$\Delta(p)\sim\frac{Z}{p^2-m^2}\Rightarrow\Delta'(p)\sim\frac{Z'}{p^2-m^2}. \tag{4.5}$$

While Z is finite due to renormalization, Z' is divergent in general.

(4) The S-matrix elements are unchanged:

$$\begin{aligned}S_n(p_1,p_2,\ldots,p_n)&=\left(\frac{1}{Z}\right)^{n/2}\lim_{p_i^2\to m^2}\left[\prod_{i=1}^{n}(p_i^2-m^2)f(p_i)\right]\\&\quad\times\langle\sigma(p_1)\cdots\sigma(p_n)\rangle^{\hat{\mathcal{L}}(\sigma,v)}\\&=\left(\frac{1}{Z'}\right)^{n/2}\lim_{p_i^2\to m^2}\left[\prod_{i=1}^{n}(p_i^2-m^2)f(p_i)\right]\\&\quad\times\langle\sigma'(p_1)\cdots\sigma'(p_n)\rangle^{\hat{\mathcal{L}}'(\sigma',v')},\end{aligned} \tag{4.6}$$

where the $f(p_i)$ are the momentum space wave functions. Since the S-matrix elements in the original theory are finite by renormalization, in the transformed theory the divergences contained in the on-shell Green functions and in Z' cancel. The transformed theory is thus "on-shell renormalizable" [10], although the Green functions themselves are divergent.

Simple proofs of these statements are given in Appendix B.

We now discuss in which sense the above general relations are valid in some expansion scheme, e.g., the loop ($\hbar$) expansion. Usually one performs the loop expansion of the effective action, or of Green functions like those in Eq. (4.2), for fixed v and v', and leaves out the tadpole graphs. In the end, one substitutes the solutions of Eqs. (2.3) and (3.6) for v and v'. The solutions of these nonlinear equations, however, involve all powers of $\hbar$, and so do the Green functions on both sides of Eq. (4.2), although they are truncated in the loop expansion for fixed v and v'. If we want to *compare* the Green functions in the old and new theories up to a given order in $\hbar$, the $\hbar$ dependence of the mean fields v and v' has to be taken into account. For example, if both sides of Eq. (4.2) are calculated to order $\hbar$ for fixed v and v', and then the solutions of Eqs. (2.3) and (3.6) are substituted for v and v', terms of order $\hbar^2$ or higher will in general be different on the left-hand side (LHS) and the RHS of Eq. (4.2). Similarly, if we calculate $\Gamma(u)$ for fixed u, and $W(J)$ for fixed J, to order $\hbar$, and insert the full solution of Eq. (2.3) for u, the quantities $\Gamma(u)$ and $W(J=0)$ differ by terms of order $\hbar^2$, and the argument given in Appendix B leading to Eq. (4.4) shows that also $\Gamma(v)$ and $\Gamma'(v')$ differ by terms of order $\hbar^2$. In particular, these higher order terms may contain divergences which are not canceled by the counterterms of the old theory.

Let us discuss some one-loop examples which demonstrate the validity of the equivalence theorem in the loop expansion, provided that the $\hbar$ dependence of v and v' is explicitly taken into account. Equation (4.5) implies the following relation between the meson self-energies in the original and transformed theories: If we write schematically $\Delta^{-1}(k^2)=k^2-m^2-\Sigma(k^2)$ in the original theory, where m^2 is the mass parameter in the Lagrangian, and $\Delta'^{-1}(k^2)=k^2-m'^2-\Sigma'(k^2)$ in the transformed theory, where $\Delta m^2\equiv m'^2-m^2\propto\hbar$ arises due to $v'\neq v$, the "pole position" up to $O(\hbar)$ is given by $m^2+\Sigma(m^2)$ in the original theory and by $m^2+\Sigma'(m^2)+\Delta m^2$ in the transformed theory. These should be the same, i.e.,

$$\Delta'^{-1}(m^2)-\Delta^{-1}(m^2)=\Sigma(m^2)-[\Sigma'(m^2)+\Delta m^2]=0. \tag{4.7}$$

Here we consider the pion propagator in free space as an example. In this case, the self-energies in the linear and nonlinear models are $\Sigma_{\pi f}=\bar{\Sigma}_{\pi f}+\delta m_\pi^2(v_f)-k^2(Z_\pi-1)$ and $\Sigma'_{\pi f}=\bar{\Sigma}'_{\pi f}-k^2(Z_\pi-1)$, where the unrenormalized self-energies $\bar{\Sigma}_{\pi f}$, $\bar{\Sigma}'_{\pi f}$ are given in Appendix C 1. The mass parameter in the nonlinear model is $\mu'^2_{\pi f}=c/v'_f\simeq c/v_f-c\Delta v/v_f^2$, where $\Delta v\equiv v'-v$. From Eq. (2.8) we have $c/v_f=-\Delta_{\pi f}^{-1}(0)=m_{\pi f}^2+\bar{\Sigma}_{\pi f}(0)+\delta m_\pi^2(v_f)$ and therefore, to $O(\hbar)$,

$$\Delta m_\pi^2\equiv\mu'^2_{\pi f}-m_{\pi f}^2=\bar{\Sigma}_{\pi f}(0)+\delta m_\pi^2(v_f)-\frac{m_{\pi f}^2}{v_f}\Delta v. \tag{4.8}$$

Then the difference between the inverse propagators becomes

$$\Delta'^{-1}_{\pi f}(k^2)-\Delta^{-1}_{\pi f}(k^2)=\bar{\Sigma}_{\pi f}(k^2)-\bar{\Sigma}_{\pi f}(0)-\bar{\Sigma}'_{\pi f}(k^2)+\frac{m_{\pi f}^2}{v_f}\Delta v. \tag{4.9}$$

The one-loop connection between v' and v follows from Eqs. (2.3) and (3.6): If we write $v=v^{(0)}+\hbar v^{(1)}$, and similar for v', and define the counterterm $u\,\delta m_\pi^2(u)$ to cancel the one-loop vacuum tadpole graphs, it follows that $v^{(0)}=v'^{(0)}=v_f$ satisfies $m_\pi^2(v_f)=c/v_f$, and

[4]Since points (3) and (4) are most relevant in free space ($\rho=T=0$), our Lorentz-covariant notation refers to this case, although the generalization is obvious. We also use the notation $\langle\sigma(p_1)\cdots\sigma(p_n)\rangle$ $(p_1+p_2+\cdots p_n=0)$ for the Fourier transform of the n-point function, with the delta function for overall momentum conservation removed.

$$\Delta v=\frac{-i}{m_{\sigma f}^2}[S'(v_f)-S(v_f)]+O(\hbar^2)=\frac{3\hbar}{2v_f}F_1(m_{\pi f}^2)+O(\hbar^2), \qquad (4.10)$$

where

$$F_1(m^2)=i\int\frac{d^4q}{(2\pi)^4}\frac{1}{q^2-m^2+i\epsilon}, \qquad (4.11)$$

and in Eq. (4.10) we used the explicit one-loop expressions for the tadpole graphs in the two theories. Inserting Eq. (4.10) into Eq. (4.9), and using the explicit forms of the self-energies given in Appendix C 1, it is easy to show that for $k^2=m_{\pi f}^2$ expression (4.9) vanishes, and therefore Eq. (4.7) is satisfied.[5]

The case of the σ meson propagator is treated similarly in Appendix C 2, and in Appendix C 3 we discuss which kinds of relations between the on-shell n-point functions are implied by Eq. (4.6), referring for definiteness to the case of the σ n-point functions. There we explicitly demonstrate the cancellation of divergences contained in the n-point function and in the Z' factors of the S matrix in the nonlinear theory, provided that the $\hbar$ dependence of v and v' is taken into account explicitly.

B. Use of the nonlinear σ model in the self-consistent mean-field approximation

In the previous subsection we have pointed out that in the loop expansion the linear and nonlinear σ models lead to the same physical results only if one really compares terms of the same order in $\hbar$, i.e., only if tadpole diagrams are counted explicitly as loop diagrams. On the contrary, if one follows the usual procedure and sums up the tadpole diagrams to all orders by minimizing the effective potential, one finds that the counterterms of the linear model are not sufficient to cancel the divergences in the nonlinear model. An example was already discussed in Sec. III, where we found that the divergence of the one-loop effective potential in the nonlinear theory cannot be absorbed by the counterterms of the linear model. Another example is shown in Eq. (C12), which implies that the term ΔG_{nt}, which represents the $O(\hbar)$ difference of the tadpole contributions in the two theories, is necessary to obtain finite on-shell Green functions (S-matrix elements) in the nonlinear theory.

An order-by-order treatment of tadpole diagrams, however, contradicts the successful mean-field theories and is clearly inadequate and impractical. We therefore conclude that, as far as one is using some expansion scheme based on self-consistent mean fields, the nonlinear σ model Lagrangian (3.4) should be considered as unrenormalizable. [We have shown this explicitly only for the loop expansion. However, also in other expansion schemes (e.g., the $1/N$ expansion [20]) the self-consistency condition for the scalar field mixes all powers of the expansion parameter.] Nevertheless, since the nonlinear σ model has definite advantages over the linear σ model, in particular for the many-body problem, one could still use Eq. (3.4) as an effective Lagrangian, and determine the necessary counterterms in each order of the expansion scheme. This method, which is somewhat similar to the treatment of loop diagrams in chiral perturbation theory [21] (although the context is quite different), will be illustrated in the next section for the case of the one-loop effective potential. (Another method would be to introduce a cutoff in the loop integrals. However, this method has the disadvantage that in many cases it leads to conflicts with symmetries.)

Before closing this section, we add two remarks: First, the "nonlinear σ model" in the common sense [22,23,4] refers to the case where ϕ is eliminated due to a constraint $\phi^2+\boldsymbol{\pi}^2=f_\pi^2$ in the original representation (2.1), leading to a nonrenormalizable Lagrangian for $\boldsymbol{\pi}$. If then one transforms [cf. Eq. (3.1)] $\boldsymbol{\pi}=\boldsymbol{\pi}'/(1+\boldsymbol{\pi}'^2/4f_\pi^2)$, the new Lagrangian is of course completely equivalent to the old one due to the equivalence theorem discussed in the previous section, since in this case there is no scalar field in the Lagrangian and hence no problem with the treatment of tadpole diagrams.[6] Second, as long as one considers only tree graphs, the Lagrangians (2.2) and (3.4) are of course completely equivalent, since the complications discussed above arise due to loop diagrams. For example, the tree graphs for π-N scattering and π-π scattering give in both representations the same well-known expressions for the s-wave scattering lengths (in units of $1/m_{\pi f}$) [24]:

$$a_{\pi N}=-\kappa\frac{g^2m_{\pi f}}{m_{Nf}}\left[\frac{m_{\pi f}^2}{4m_{Nf}^2}\frac{1}{1-m_{\pi f}^2/4m_{Nf}^2}+\frac{m_{\pi f}^2}{m_{\sigma f}^2}\right], \qquad (4.12)$$

$$a_{\pi\pi}=\frac{m_{\pi f}^2}{32\pi f_\pi^2}\left[7+\frac{27m_{\pi f}^2}{m_{\sigma f}^2-4m_{\pi f}^2}+\frac{2m_{\pi f}^2}{m_{\sigma f}^2}\right]. \qquad (4.13)$$

Here $\kappa=[4\pi(1+m_{\pi f}/m_{Nf})]^{-1}$, and $a_{\pi N}(a_{\pi\pi})$ refers to isospin zero in the t (s) channel. In the nonlinear σ model, the first term in Eq. (4.12) is due to the nucleon pole Born graphs, and the second term due to σ meson exchange. Similarly, the first term in Eq. (4.13) is due to the $\boldsymbol{\pi}^4$ contact term, while the rest is due to σ meson exchange. Both expressions reproduce the current algebra predictions in the limit $m_\sigma\to\infty$. (The experimental values are $a_{\pi N}=-0.015\pm0.015$ [25] and $a_{\pi\pi}=0.26\pm0.05$ [26].)

V. EOS IN A ONE-LOOP CALCULATION

To illustrate the use of the nonlinear chiral σ model for nuclear matter, in this section we will calculate the one-loop effective potential and the resulting EOS at finite density and temperature.

In addition to the counterterms of the linear model in-

[5] Of course, Eq. (4.9) can also be derived directly from the Weinberg transformation, i.e., the analog of Eq. (4.2) for the pion. This is also shown in Appendix C 1.

[6] In two dimensions, the Lagrangian in the original representation is manifestly renormalizable, and the one in the new representation is "on-shell renormalizable" in the sense of the equivalence theorem [22].

cluded in Eq. (2.6), a new counterterm is needed to renormalize the pion loop contribution (3.8). It has the form[7]

$$\delta\mathcal{L}=-\frac{c^2}{\phi^2}\delta\alpha=-\frac{c^2}{u^2}\delta\alpha\frac{1}{(1+\sigma/u)^2}, \qquad (5.1)$$

and is of second order in the chiral-symmetry-breaking parameter c. If $\delta\alpha$ is chosen such that the new term in the effective potential $[\delta V_{\rm eff}=(c^2/u^2)\delta\alpha]$ cancels the divergence in Eq. (3.8), it is clear that also the σ^n counterterms, which are obtained by expanding Eq. (5.1) in powers of σ, cancel the divergences of the one-pion-loop contribution to the σ^n Green functions at zero external momenta. To determine also the finite part of $\delta\alpha$, we will require that the term $\propto\sigma$ in Eq. (5.1) cancel the whole pion loop contribution to the σ one-point function in free space ($u=v_f$). This gives

$$\delta\alpha=\frac{3\hbar}{64\pi^2}\left[K-\ln\mu_{\pi f}^2-\frac{1}{2}\right], \qquad (5.2)$$

$$\left[\bar{V}_{F,\pi}+\frac{c^2}{u^2}\delta\alpha\right]_{u=v}=\frac{3\hbar}{64\pi^2}\left[\mu_\pi^4\ln\frac{\mu_\pi^2}{\mu_{\pi f}^2}-\frac{1}{2}(\mu_\pi^4-\mu_{\pi f}^4)\right]. \qquad (5.3)$$

The remaining counterterms $\delta\mu^2$ and $\delta\lambda^2$ are determined in the usual way [5] by requiring that, in an expansion of the effective potential at $\mu_c=T=0$ around $u=v_f$, the first two terms are solely due to the classical potential $V_{\rm cl}$, i.e., that there are no loop contributions to the one- and two-point σ Green functions in free space at zero momentum:

$$\left.\frac{\partial V_{\rm eff}}{\partial u}\right|_{\mu_c=T=0,u=v_f}=v_f\,m_{\pi f}^2-c, \qquad (5.4)$$

$$\left.\frac{\partial^2 V_{\rm eff}}{\partial u^2}\right|_{\mu=T=0,u=v_f}=m_{\sigma f}^2, \qquad (5.5)$$

where, as in Sec. II, $m_{\pi f}^2=\mu^2+\lambda^2 v_f^2$ and $m_{\sigma f}^2=\mu^2+3\lambda^2 v_f^2$. As a result of the condition (5.4), the pion mass parameters in free space are the same for the linear and nonlinear models ($\mu_{\pi f}^2=m_{\pi f}^2$), since in free space v_f is determined such that the RHS of Eq. (5.4) vanishes.

According to the above procedure, the renormalized Feynman part of the effective potential $V_{\rm eff}(u)=V_{\rm cl}+V_F+V_D$ in the nonlinear model at $u=v$ becomes $V_F=V_{FN}+V_{F\sigma}+V_{F\pi}$, where

$$V_{FN}=-\frac{1}{8\pi^2}\left[m_N^4\ln\frac{m_N^2}{m_{Nf}^2}-\frac{3}{2}(m_N^4-m_{Nf}^4)+2m_{Nf}^2(m_N^2-m_{Nf}^2)\right], \qquad (5.6)$$

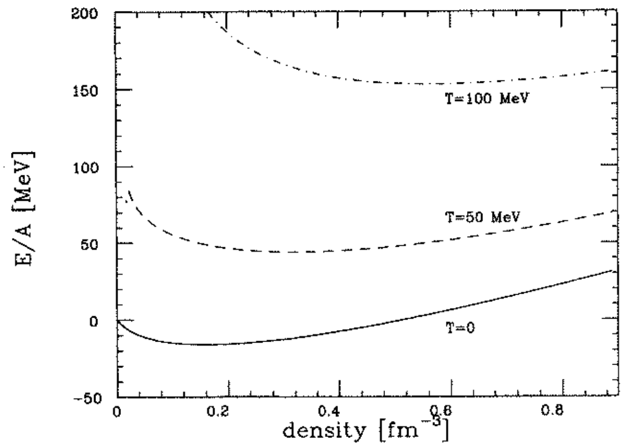

FIG. 1. The binding energy per nucleon obtained in the one-loop approximation to the nonlinear σ model as a function of the density for three temperatures $T=0$ (solid line), $T=50$ MeV (dashed line), and $T=100$ MeV (dot-dashed line). The parameters used are $m_\sigma=600$ MeV, $m_\omega=783$ MeV, $g=10$, and $g_\omega=5$.

$$V_{F\sigma}=\frac{1}{64\pi^2}\left[m_\sigma^4\ln\frac{m_\sigma^2}{m_{\sigma f}^2}-\frac{3}{2}(m_\sigma^4-m_{\sigma f}^4)+2m_{\sigma f}^2(m_\sigma^2-m_{\sigma f}^2)\right], \qquad (5.7)$$

$$V_{F\pi}=\frac{3}{64\pi^2}\left[\mu_\pi^4\ln\frac{\mu_\pi^2}{\mu_{\pi f}^2}-\frac{1}{2}(\mu_\pi^4-\mu_{\pi f}^4)-\frac{1}{4}\mu_{\pi f}^4\left(\frac{v^2}{v_f^2}-1\right)^2\right]. \qquad (5.8)$$

The forms (5.6) and (5.7) are the same as in the linear σ model [5,12]. Including the Hartree term due to the ω meson, which involves the mean field w_0, in the classical part $V_{\rm cl}$, and expressing the parameters μ^2 and λ^2 by $m_{\pi f}^2$ and $m_{\sigma f}^2$, we obtain, after subtracting the vacuum value [5],

$$V_{\rm cl}=\frac{m_{\pi f}^2}{2}(v^2-v_f^2)+\frac{m_{\sigma f}^2-m_{\pi f}^2}{8v_f^2}(v^2-v_f^2)^2-c(v-v_f)-\frac{1}{2}m_{\omega f}^2 w_0^2. \qquad (5.9)$$

The form of the μ_c,T-dependent part V_D is given in Appendix D. The mean fields v and w_0 are determined by minimizing the effective potential for fixed μ_c,T. For w_0 this gives $w_0=g_\omega\rho/m_{\omega f}^2$ with the baryon density $\rho=\partial V_{\rm eff}/\partial\mu_c$. The pressure and energy density of the system are determined from the thermodynamic potential $\Omega/V=V_{\rm eff}(v)$ by the standard thermodynamic relationships.

In the numerical calculations we use $v_f=f_\pi$, and therefore $g=m_{Nf}/v_f\simeq 10$. Fixing also $m_\omega=783$ MeV, the free parameters are m_σ and g_ω, which can be fitted to the empirical saturation point of the energy per nucleon at $T=0$. This gives $m_\sigma\simeq 600$ MeV, $g_\omega\simeq 5$. We note that this value of m_σ is not inconsistent with the ones extracted from recent $\pi\pi$ phase shift analyses [27].

In Figs. 1 and 2 we show the binding energy per nucleon and the pressure for $T=0$, 50, and 100 MeV as functions of the density. One finds that the effect of the pion loop term on these bulk properties is negligibly small. This is natural,

[7]Since in the following we consider only the nonlinear model, we omit the primes on all quantities ($v'\to v$, $\phi'\to\phi$, $m_N'\to m_N$, etc.). However, we still distinguish the pion mass parameter in the nonlinear model (μ_π^2) from the one in the linear model (m_π^2).

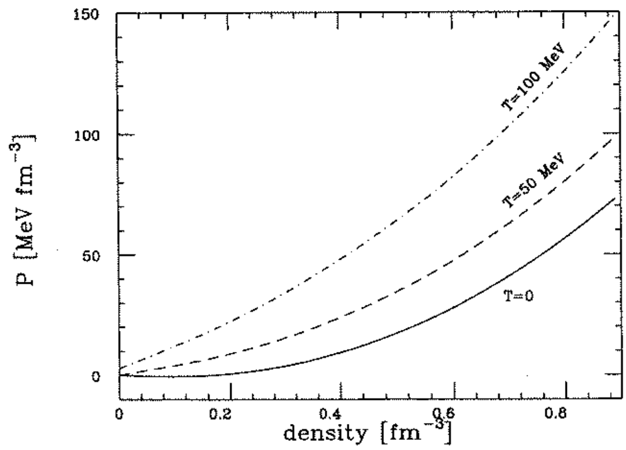

FIG. 2. Same as Fig. 1 for the pressure.

since we are including here only the effect of the s-wave πN interaction, which is known to be weak. Therefore, the EOS is practically the same as the one obtained in Refs. [5,28] on the basis of the linear σ model by leaving out the (tachyonic) pion loop contribution. It is known that this EOS is very soft compared to other relativistic EOS. This is indicated by the slow increase of the energy and the pressure at high densities, and the low incompressibility of about 130 MeV at $T=0$.

In Fig. 3 we show the effective masses m_N, m_σ, and μ_π as functions of the density for the same values of the temperatures as used in Figs. 1 and 2. As we mentioned already in Sec. II, in order to investigate the phase transition to the Wigner mode ($v=0$), one should include the ring-type diagrams to avoid the tachyon pole in the σ propagator. However, we see from Fig. 3 that, due to the rather slow decrease of v with increasing density, we are far from the tachyonic region even at high densities.

In order to elucidate the effect of the enhanced pion mass, we finally consider the effective πN s-wave scattering

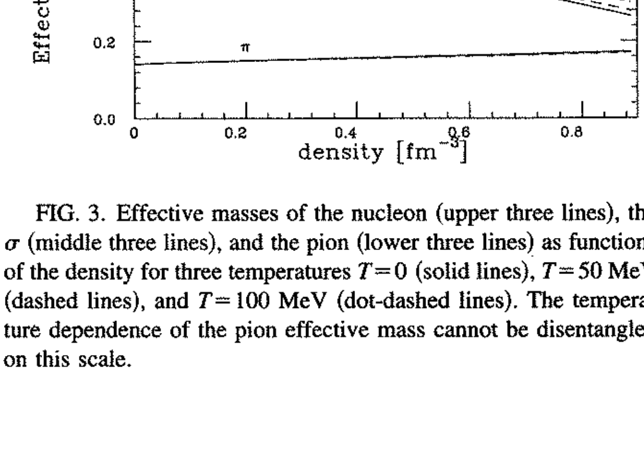

FIG. 3. Effective masses of the nucleon (upper three lines), the σ (middle three lines), and the pion (lower three lines) as functions of the density for three temperatures $T=0$ (solid lines), $T=50$ MeV (dashed lines), and $T=100$ MeV (dot-dashed lines). The temperature dependence of the pion effective mass cannot be disentangled on this scale.

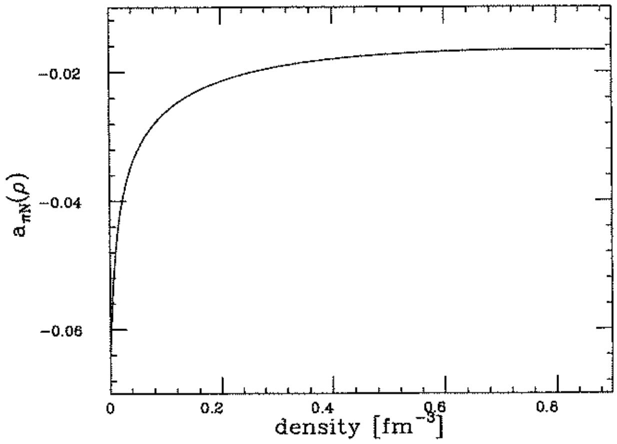

FIG. 4. The effective πN scattering length in units of $m_{\pi f}^{-1}$ for $T=0$ as a function of the density. The empirical values for zero density and normal nuclear matter density are -0.015 and -0.03, respectively.

length in the medium $\tilde{a}_{\pi N}(\rho)$ (in units of $m_{\pi f}^{-1}$), which is defined via the depth of the s-wave πN optical potential $V_{\rm opt}$ at threshold by [29,12]

$$2m_{\pi f}V_{\rm opt}(E=m_{\pi f})=-\kappa^{-1}\frac{\rho}{m_{\pi f}}\tilde{a}_{\pi N}(\rho), \quad (5.10)$$

where κ is defined below Eq. (4.13). The empirical value of $\tilde{a}_{\pi N}(\rho)$ at normal nuclear matter density is -0.03 [29]. Compared to the zero density value -0.015, this implies some additional repulsion due to the presence of the nuclear medium. Since the quantity on the LHS of Eq. (5.10) is given in terms of the pion propagator by $-\Delta_\pi^{-1}(q_0=m_{\pi f},\mathbf{q}=0)$, we have, in the Hartree approximation,

$$\tilde{a}_{\pi N}(\rho)=-\kappa m_{\pi f}\frac{\mu_\pi^2-m_{\pi f}^2}{\rho} \quad (5.11)$$

$$\overset{(\rho\to 0)}{\longrightarrow}-\kappa\frac{cm_{\pi f}}{v_f^2}\frac{\partial v}{\partial\rho}\bigg|_{\rho=0}=-\kappa\frac{g^2}{m_{Nf}}\frac{m_{\pi f}^3}{m_{\sigma f}^2}. \quad (5.12)$$

To obtain the zero density limit, we used the relation $\partial v/\partial\rho|_{\rho=0}=-g/m_\sigma^2$, which follows from the more general relation given in Ref. [5] [see Eq. (3.22a) of Ref. [5]]. By comparing with Eq. (4.12) and the discussion given there, we see that in the zero density limit the Hartree expression for $\tilde{a}_{\pi N}$ reduces to the σ meson pole contribution to the πN scattering length. [The nucleon pole contribution arises from the one-loop graphs in the pion self-energy. Numerically, however, the first term in Eq. (4.12) is smaller by a factor of $\simeq(m_{\sigma f}/2m_{Nf})^2\simeq 0.1$ for the σ mass used here.]

The effective πN scattering length (5.11) is shown for $T=0$ in Fig. 4 as a function of the density. We see that at the saturation density the result is roughly consistent with the empirical value. The zero density value, however, is much larger in magnitude than the experimental value due to the rather low σ mass used here. (To reproduce the experimental zero density value, one would need a σ mass of about 1.5 GeV.)

VI. SUMMARY AND CONCLUSIONS

At present the chiral σ model in the linear representation does not seem to be well suited for the nuclear many-body problem due to the appearance of a tachyon pole in the Hartree pion propagator. Therefore, in this paper we investigated the nonlinear chiral σ model as a candidate to include both the pionic degrees of freedom and vacuum fluctuation effects in the description of many-nucleon systems. The square of the in-medium pion mass in this model is positive definite, and therefore the nonlinear model in the Hartree approximation represents a more natural starting point for the many-body problem than the linear model.

Since the Lagrangian of the nonlinear model can be obtained from the one of the linear model by a field transformation, one might expect that the two models are equivalent to each other as far as physical quantities are concerned. In this paper, however, we have shown that in any expansion scheme which is based on self-consistent scalar mean fields, the equivalence between the two models does not hold. This is due to the fact that the self-consistency conditions, which are different in the two models, bring in all powers of the expansion parameter, while the equivalence theorem holds only if we compare terms which are of the *same* order. In particular, we have shown that if one employs self-consistent scalar fields, the counterterms introduced in the linear model are insufficient to cancel the divergences in physical quantities in the nonlinear model; i.e., the "on-shell renormalizability" [10] is lost. This point was illustrated explicitly by using various one-loop examples, like the effective potential, pole positions of meson propagators, and S-matrix elements.

Based on this observation, one still can employ the nonlinear σ model as a nonrenormalizable effective model, and introduce the necessary counterterms in each order of the expansion scheme, similar in spirit to the treatment of loops in chiral perturbation theory [21], although the context is different. We illustrated this procedure for the calculation of the one-loop effective potential, and discussed the resulting EOS. As compared to the linear model, the new ingredient is the pion loop contribution, which arises due to the (slightly) enhanced pion mass in the medium. For the bulk properties, however, this contribution turns out to be negligibly small. To illustrate the effect of the enhanced pion mass in the medium more clearly, we considered the pion optical potential in the Hartree approximation. We pointed out that its size at normal nuclear matter density is consistent with the empirical depth of the optical potential. The zero density limit of the resulting effective scattering length, however, turns out to be larger in magnitude than the experimental one due to the rather low σ mass used to fit the saturation point of nuclear matter. These numerical examples at least indicate that the Hartree approximation to the nonlinear model has no obvious difficulties to describe nuclear matter properties.

ACKNOWLEDGMENTS

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APPENDIX A: CHIRAL TRANSFORMATIONS IN THE NONLINEAR REPRESENTATION

Here we briefly discuss the chiral transformation properties of the fields in the nonlinear Lagrangian (3.3) and the role of the Jacobian term. From the well-known infinitesimal chiral transformations in the linear representation and Eq. (3.1) it follows that the new pion field transforms as $\xi_i \to \xi_i - (\alpha_i/2)(1-\xi^2) - \xi_i(\alpha\cdot\xi)$, which is a special case of the general nonlinear transformation derived in [7]. Accordingly, the transformation of the fields $B \equiv (N,(D^\mu N)$, or $(D^\mu \xi))$ has the form of a local isospin transformation:

$$B \to B + iT\cdot(\alpha\times\xi)B, \tag{A1}$$

with $T=\tau/2$ for $B=(N,(D^\mu N))$ and $(T_i)_{jk} = -i\epsilon_{ijk}$ for $B=(D^\mu\xi)$. From this it follows that any isoscalar function consisting of $N,(D^\mu N),(D^\mu\xi)$, and ϕ' is chiral invariant.

Concerning the determinant (3.2), we note that, first, the quantity A is just the square root of the coefficient of the kinetic term for the ξ field in Eq. (3.3). If we would start directly from the nonlinear σ model Lagrangian, express the partition function in "Hamiltonian form," and integrate out the momentum conjugate to ξ, we would just get the same factor J of Eq. (3.2) [30]. Thus, although we derived J as a Jacobian of a field transformation, it should be there even if one starts directly from the nonlinear σ model. Second, the determinant is necessary to make the measure of the partition function of the nonlinear model chiral invariant: From the transformation properties discussed above it follows that $(\partial_\mu\xi_i)A^2_{ij}(\partial^\mu\xi_j)$ is chiral invariant. Therefore, the measure $\mathcal{D}\xi\,\det(A_{ij})$, rather than $\mathcal{D}\xi$, is chiral invariant. This has consequences, for example, for the one-loop pion self-energy given in Appendix C 1, where the ghost loop (last diagram in Fig. 5) cancels the contribution of the pion loop (fourth diagram in Fig. 5) at zero external momenta.

APPENDIX B: THE EQUIVALENCE THEOREM

In this appendix we briefly indicate the proofs of the statements (1)–(4) of Sec. IV A [9,10].

The generating functionals in the linear and nonlinear theories are given by[8]

$$Z(J) = e^{iW(J)} = \int \mathcal{D}\phi \exp\left(i\int [\mathcal{L}(\phi)+J\phi]\right) = \int \mathcal{D}\phi' \exp\left(i\int \{\mathcal{L}'(\phi')+J\phi[\phi']\}\right), \tag{B1}$$

[8]In this as well as the following appendixes we do not indicate $\hbar$ explicitly. The integrals in the exponents of Eqs. (B1)–(B4), as well as those in Eqs. (B7) and (B8), refer to four-dimensional space-time. We also note that the quantities defined in Eqs. (B1) and (B3) are related by $\tilde{Z}(J,u) = \exp(-i\int Ju)Z(J)$ and $\tilde{W}(J,u) = W(J) - \int Ju$, and similarly for the transformed theory.

$$Z'(J)=e^{iW'(J)}=\int \mathcal{D}\phi' \exp\left(i\int [\mathcal{L}'(\phi')+J\phi']\right), \quad \text{(B2)}$$

and the generating functionals for the shifted theories are [31]

$$\tilde{Z}(J,u)=e^{i\tilde{W}(J,u)}=\int \mathcal{D}\sigma \exp\left(i\int [\hat{\mathcal{L}}(\sigma,u)+J\sigma]\right)$$
$$=\int \mathcal{D}\sigma' \exp\left(i\int \{\hat{\mathcal{L}}'(\sigma',u') + J(\phi[\sigma'+u']-u)\}\right), \quad \text{(B3)}$$

$$\tilde{Z}'(J,u')=e^{i\tilde{W}'(J,u')}=\int \mathcal{D}\sigma' \exp\left(i\int [\hat{\mathcal{L}}'(\sigma',u')+J\sigma']\right). \quad \text{(B4)}$$

In the last equalities of Eqs. (B1) and (B3), the variables ϕ' and σ' were introduced according to $\phi=\phi[\phi']$ and $\sigma+u=\phi[\sigma'+u']$. The corresponding mean fields are given by

$$v=\langle\phi\rangle^{\mathcal{L}(\phi)}/Z(0)=\partial W/\partial J|_{J=0}, \quad \text{(B5)}$$

$$v'=\langle\phi'\rangle^{\mathcal{L}'(\phi')}/Z(0)=\partial W'/\partial J|_{J=0}, \quad \text{(B6)}$$

where we used $Z(0)=Z'(0)$. By taking derivatives with respect to J at $J=0$ of Eq. (B1), the relation (4.1) is obtained. In a similar manner, from Eq. (B3) for $u=v$ and $u'=v'$ we obtain Eq. (4.2) with $\sigma[\sigma',v']$ given by Eq. (4.3).

Relation (4.4) follows simply from the definition of the effective action, i.e.,

$$\Gamma(u)=\tilde{W}(J(u),u)=W(J(u))-\int J(u)u, \quad \left(\frac{\partial W}{\partial J}\bigg|_{J=J(u)}=u\right), \quad \text{(B7)}$$

$$\Gamma'(u)=\tilde{W}'(J(u),u)=W'(J'(u))-\int J'(u)u, \quad \left(\frac{\partial W'}{\partial J}\bigg|_{J=J'(u)}=u\right), \quad \text{(B8)}$$

where the functions $J(u)$, $J'(u)$ obey the relations given in the brackets above. Comparison with Eq. (B5), Eq. (B6) shows that v and v' are solutions of $J(u)=0$ and $J'(u)=0$, respectively [see also Eqs. (2.3) and (3.6)], and therefore $\Gamma(v)=W(0)$, $\Gamma'(v')=W'(0)=W(0)$, where the last identity follows from Eqs. (B1) and (B2). This gives Eq. (4.4).

To show Eq. (4.5), one uses the fact that for canonical transformations [4] $\sigma=\sigma'+f(\sigma',v')+C(v')$, where f is second or higher order in σ', and $C(v')$ is some function of v'. Then Eq. (4.2) for the two-point function gives

$$\langle\sigma\sigma\rangle^{\hat{\mathcal{L}}(\sigma,v)}=\langle\sigma'\sigma'\rangle^{\hat{\mathcal{L}}'(\sigma',v')}+\langle\sigma'f\rangle^{\hat{\mathcal{L}}'(\sigma',v')} +\langle f\sigma'\rangle^{\hat{\mathcal{L}}'(\sigma',v')}+\langle ff\rangle^{\hat{\mathcal{L}}'(\sigma',v')} +[\text{one-point functions}+C^2(v')]. \quad \text{(B9)}$$

If $D(p^2)$ is the σ-irreducible σ'-f vertex function and $E(p^2)$ the σ-irreducible f-f vertex function, we can write $\langle\sigma'(p)\,f(-p)\rangle^{\hat{\mathcal{L}}'(\sigma',v')} = D(p^2)\langle\sigma'(p)\,\sigma'(-p)\rangle^{\hat{\mathcal{L}}'(\sigma',v')}$ and $\langle f(p)f(-p)\rangle^{\hat{\mathcal{L}}'(\sigma',v')}=E(p^2)+D^2(p^2)\times\langle\sigma'(p)\sigma'(-p)\rangle^{\hat{\mathcal{L}}'(\sigma',v')}$, and Eq. (B9) leads to

$$Z=\lim_{p^2\to m^2}(p^2-m^2)\{\Delta'(p)[1+D(p^2)]^2+E(p^2)\}. \quad \text{(B10)}$$

In the absence of bound states (in particular, in perturbation theory), the functions $D(p^2)$ and $E(p^2)$ satisfy the usual dispersion relations without pole terms. It then follows that $\Delta'(p)$ must have a pole at the same position as $\Delta(p)$, and the residues are related by

$$Z'=\frac{Z}{[1+D(m^2)]^2}. \quad \text{(B11)}$$

To show the equality of S-matrix elements, one uses $\sigma=\sigma'+f(\sigma',v')+C(v')$ in the second line of Eq. (4.6) and notes that, in order to have n σ poles of the Green function, every external block $f(p)$ must lead to an intermediate σ' propagator; i.e., we can replace $f(p_i)\to D(p_i^2)\sigma'(p_i)$. Moreover, terms involving $C(v')$ give only contributions with less that n sigma poles, and therefore we have effectively $\sigma(p_i)\to[1+D(m^2)]\sigma'(p_i)$ at the pole. This gives Eq. (4.6) with Z' given by Eq. (B11).

APPENDIX C: ONE-LOOP EXAMPLES FOR THE EQUIVALENCE THEOREM

In this appendix we give various one-loop examples for the general discussions on the equivalence theorem presented in Sec. IV.[9]

1. Pion propagator

In order to verify that the RHS of Eq. (4.9) vanishes for $k^2=m_\pi^2$, we need the forms for the unrenormalized self-energies which follow from the Feynman diagrams shown in Fig. 5:

$$\bar{\Sigma}_\pi(k^2)=\lambda^2[F_1(m_\sigma^2)+5F_1(m_\pi^2)+4\lambda^2v^2F_2(k^2,m_\pi^2,m_\sigma^2)] -8g^2F_1(m_N^2)+4g^2k^2F_2(k^2,m_N^2,m_N^2), \quad \text{(C1)}$$

[9]Since all formulas of this appendix refer to free space ($\mu_c=T=0$), we leave out the index f on all quantities, i.e., $v_f\to v$, $m_{Nf}\to m_N$, etc.

$$\bar{\Sigma}'_\pi(k^2)=\frac{1}{v^2}\left[\left(\frac{k^2}{2}+m_\sigma^2\right)F_1(m_\pi^2)+(2k^2-m_\sigma^2-m_\pi^2)F_1(m_\sigma^2)+(k^2-m_\sigma^2)^2F_2(k^2,m_\pi^2,m_\sigma^2)\right]+4g^2k^2F_2(k^2,m_N^2,m_N^2). \quad \text{(C2)}$$

Since we calculate up to one-loop order, we have set $v'=v$ and $\mu_\pi'^2=m_\pi^2$ in the self-energies. F_1 has been defined in Eq. (4.11), and

$$F_2(k^2,m_1^2,m_2^2)=i\int\frac{d^4q}{(2\pi)^4}\times\frac{1}{(q^2-m_1^2+i\epsilon)[(k-q)^2-m_2^2+i\epsilon]}. \quad \text{(C3)}$$

It is easy to verify that for $k^2=0$ the expression (C2) is finite. Using $2\lambda^2v^2=m_\sigma^2-m_\pi^2$, it is also easy to see that the RHS of Eq. (4.9) vanishes for $k^2=m_\pi^2$.

We can also verify that Eq. (4.9) follows directly from the Weinberg transformation, i.e., the analog of Eq. (B9) for the pion. For this, we note that to one-loop order the transformation of the pion field can be approximated as $\pi=\pi'+f_\pi$, where $f_\pi=\pi'(\sigma'/v-\pi'^2/4v^2)$. If $D_\pi(k^2)$ is the pion-irreducible π'-f_π vertex, and $E_\pi(k^2)$ the pion-irreducible f_π-f_π vertex, the Weinberg transformation gives the connection

$$\Delta_\pi(k^2)=\Delta'_\pi(k^2)[1+D_\pi(k^2)]^2+E_\pi(k^2), \quad \text{(C4)}$$

which can be rewritten in the following form valid to $O(\hbar)$:

$$\Delta_\pi^{-1}(k^2)-\Delta_\pi'^{-1}(k^2)=(k^2-m_\pi^2)2D_\pi(k^2)+(k^2-m_\pi^2)^2E_\pi(k^2). \quad \text{(C5)}$$

The one-loop expressions of the vertices $D_\pi(k^2)$ and $E_\pi(k^2)$ are

$$D_\pi(k^2)=\frac{-1}{v^2}\left[\frac{1}{4}F_1(m_\pi^2)+F_1(m_\sigma^2)+(k^2-m_\sigma^2)F_2(k^2,m_\pi^2,m_\sigma^2)\right], \quad \text{(C6)}$$

$$E_\pi(k^2)=\frac{-1}{v^2}F_2(k^2,m_\pi^2,m_\sigma^2). \quad \text{(C7)}$$

With these forms, the equivalence of Eqs. (4.9) and (C5) can be easily verified.

2. Sigma propagator

To show the validity of Eq. (4.7) for the σ propagator, we use the fact that the σ self-energies in the linear and nonlinear models differ only due to the pion loop and ghost (g) loop contributions, which are obtained from the Feynman diagrams of Fig. 6 as

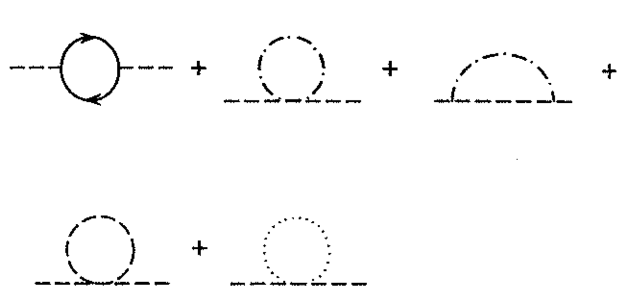

FIG. 5. Feynman diagrams for the one-loop pion self-energy. The solid, dashed, dot-dashed, and dotted lines represent the nucleon, pion, σ, and ghost propagators, respectively. The last diagram (ghost loop) is present only in the nonlinear model.

$$\bar{\Sigma}_\sigma^{\pi\ \mathrm{loop}}(k^2)=3\lambda^2[F_1(m_\pi^2)+2\lambda^2v^2F_2(k^2,m_\pi^2,m_\pi^2)], \quad \text{(C8)}$$

$$\bar{\Sigma}_\sigma'^{(\pi+g)\ \mathrm{loop}}(k^2)=\frac{3}{v^2}\left[-(k^2-m_\pi^2)F_1(m_\pi^2)+\frac{1}{2}(k^2-m_\pi^2)^2F_2(k^2,m_\pi^2,m_\pi^2)\right]. \quad \text{(C9)}$$

Note that for $k=0$ these expressions agree with the second derivatives of the unrenormalized pion loop terms in the effective potentials [see Eqs. (2.6) and (3.8)] with respect to u at $u=v$. The mass shift Δm_σ^2 is given by $\Delta m_\sigma^2=3\lambda^2(v'^2-v^2)=9\lambda^2F_1(m_\pi^2)+O(\hbar^2)$, where we used Eq. (4.10). Therefore the difference between the inverse propagators becomes, to one-loop order,

$$\Delta_\sigma^{-1}(k^2)-\Delta_\sigma'^{-1}(k^2)=\frac{-3}{v^2}F_1(m_\pi^2)(k^2-m_\sigma^2)+\frac{3}{2v^2}F_2(k^2,m_\pi^2,m_\pi^2)\times[(k^2-m_\pi^2)^2-(m_\sigma^2-m_\pi^2)^2]. \quad \text{(C10)}$$

Setting $k^2=m_\sigma^2$, we see from this that Eq. (4.7) is satisfied, and we also obtain the expression for $D_\sigma(k^2)$ [see Eq. (B11)]:

$$D_\sigma(k^2)=\frac{3}{2v^2}[F_1(m_\pi^2)-(k^2-m_\pi^2)F_2(k^2,m_\pi^2,m_\pi^2)]. \quad \text{(C11)}$$

This coincides with the expression obtained for the σ-irreducible σ'-f_σ vertex, where to one-loop order

$$f_\sigma\simeq-\frac{\pi'^2}{2v}\left(1+\frac{\sigma'}{v}\right).$$

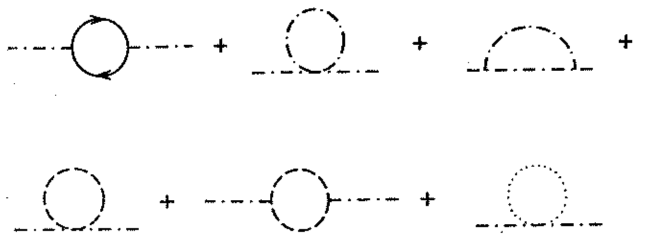

FIG. 6. Feynman diagrams for the one-loop σ self-energy. The solid, dashed, dot-dashed, and dotted lines represent the nucleon, pion, σ, and ghost propagators, respectively. The last diagram (ghost loop) is present only in the nonlinear model.

3. σ^3 S matrix

We first discuss which kind of relations between the on-shell *amputated* n-point functions (G_n) are implied by Eq. (4.6), referring for definiteness to the σ n-point functions. Equations (4.6) and (B11) imply that to one-loop order $G'_n=(Z/Z')^{n/2}G_n \simeq G_{n,t}(1+nD_\sigma)+G_{n,\ell}$, where we split G_n in the linear model into a tree graph $G_{n,t}$ and a loop contribution $G_{n,\ell}$, and $D_\sigma \equiv D_\sigma(k^2=m_\sigma^2)$. The tree graph in the nonlinear model $G'_{n,t}$ differs from the one in the linear model due to $v' \neq v$. We therefore obtain the relation

$$G'_{n,\ell}-G_{n,\ell}+\Delta G_{n,t}=nG_{n,t}D_\sigma, \tag{C12}$$

where $\Delta G_{n,t} \equiv G'_{n,t}-G_{n,t}$. In particular, for $n>4$ there are no tree graphs, and the on-shell n-point functions are identical (and finite).

Let us verify Eq. (C12) explicitly for the case $n=3$. The one-loop Feynman graphs for the σ^3 off-shell Green function G_3 are shown in Fig. 7. The tree graph is $G_{3,t}=-6i\lambda^2 v$, and using Eq. (4.10) we obtain

$$\Delta G_{3t}=-6i\lambda^2(v'-v)=\frac{-9i}{2v^3}(m_\sigma^2-m_\pi^2)F_1(m_\pi^2). \tag{C13}$$

Since the loop contributions in the two models differ only due to the pion and ghost loops, we give the expressions only for these contributions. In the linear model we obtain

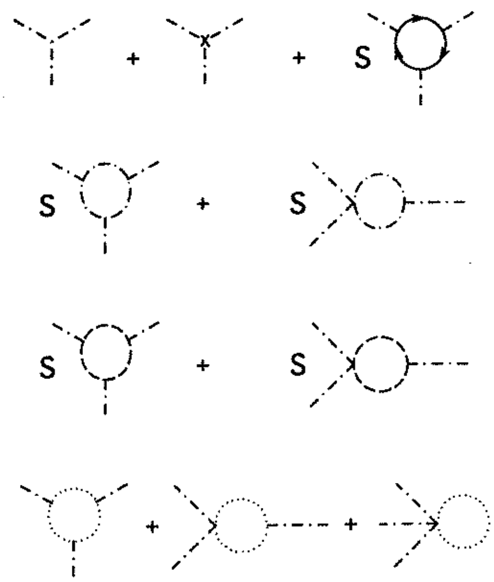

FIG. 7. Feynman diagrams for the σ^3 Green functions up to one-loop order. The solid, dashed, dot-dashed, and dotted lines represent the nucleon, pion, σ, and ghost propagators, respectively. The first diagram is the tree graph, and the second one is a counterterm $\propto \delta\lambda^2$. The last three diagrams (ghost loops) are present only in the nonlinear model. The symbol S denotes the symmetrizer with respect to the external momenta k_1, k_2, k_3.

$$G_{3\ell}^{\pi\,\mathrm{loop}}(k_1,k_2,k_3)=\frac{-3i}{2v^2}\{(m_\sigma^2-m_\pi^2)^2[F_2(k_1^2)+F_2(k_2^2)+F_2(k_3^2)]+(m_\sigma^2-m_\pi^2)^3\times[F_3(k_1,k_2)+F_3(k_2,k_1)]\}, \tag{C14}$$

where we use $F_2(k^2)\equiv F_2(k^2,m_\pi^2,m_\pi^2)$, and the "triangle graphs" are defined by

$$F_3(k_1,k_2)=i\int\frac{d^4q}{(2\pi)^4}\frac{1}{[q^2-m_\pi^2+i\epsilon][(q+k_1)^2-m_\pi^2+i\epsilon][(q+k_1+k_2)^2-m_\pi^2+i\epsilon]}. \tag{C15}$$

It is easy to see that for $k_i=0$ $(i=1,2,3)$, Eq. (C14) agrees with $-i$ times the third derivative of the unrenormalized pion loop term in Eq. (2.6) with respect to u at $u=v$.

In the nonlinear model, owing to the derivative couplings, one obtains an expression for the loop graphs which involves the integration momentum (q) also in the numerator. By using relations like $2q\cdot(q+k)=(q^2-m_\pi^2)+[(q+k)^2-m_\pi^2]-(k^2-m_\pi^2)+m_\pi^2$ and shifts of the integration variable, one can remove the q dependence from the numerators with the result

$$G_{3\ell}'^{(\pi+g)\,\mathrm{loop}}(k_1,k_2,k_3)=\frac{-3i}{2v^2}\{2F_1(k_1^2+k_2^2+k_3^2-3m_\pi^2)-F_2(k_1^2)(k_1^2-m_\pi^2)(k_2^2+k_3^2-2m_\pi^2)-F_2(k_2^2)(k_2^2-m_\pi^2)(k_3^2+k_1^2-2m_\pi^2)-F_2(k_3^2)(k_3^2-m_\pi^2)(k_1^2+k_2^2-2m_\pi^2)+(k_1^2-m_\pi^2)(k_2^2-m_\pi^2)(k_3^2-m_\pi^2)[F_3(k_1,k_2)+F_3(k_2,k_1)]\}, \tag{C16}$$

with $F_1 \equiv F_1(m_\pi^2)$. For $k_i=0$ ($i=1,2,3$), this agrees with $-i$ times the third derivative of the pion loop term in Eq. (3.8) with respect to u at $u=v$.

From Eqs. (C11), (C13), (C14), and (C16) we see that relation (C12) for $n=3$ is valid for the on-shell case ($k_i^2=m_\sigma^2$).

APPENDIX D: THE FORM OF V_D

The explicitly μ_c, T-dependent part of the effective potential has the form

$$V_D(v) = -\frac{4}{3}\int \frac{d^3k}{(2\pi)^3}\frac{k^2}{E(k)}[f_-(k)+f_+(k)] - \frac{1}{3}\sum_{i=\sigma,\omega,\pi} a_i \int \frac{d^3k}{(2\pi)^3}\frac{k^2}{\omega_i(k)} f_i(k), \quad \text{(D1)}$$

where $a_\sigma=1$, $a_\omega=a_\pi=3$, $E(k)=\sqrt{k^2+m_N^2}$, $\omega_\sigma=\sqrt{k^2+m_\sigma^2}$, $\omega_\omega=\sqrt{k^2+m_{\omega f}^2}$, $\omega_\pi=\sqrt{k^2+\mu_\pi^2}$, and the distribution functions $f_i=[\exp\{\beta\omega_i(k)\}-1]^{-1}$ and $f_\pm(k)=[\exp\{\beta(E(k)\pm\mu_c^*)\}+1]^{-1}$. Here $\mu_c^*=\mu_c-g_\omega w^0$. We note that all masses which appear in Eq. (D1) are in-medium masses as defined in the main text, except for the ω meson mass.

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